

Mathematics (Hons.) Paper-I (Arts/Sc.)

Answer any six questions.

1. Define a countable set and prove that countable union of countable sets is countable.
2. (a) Define a partial order relation on a set and illustrate the concept with two examples.
(b) Define a total order relation on a set and show by an example that a partially ordered set may fail to be totally ordered.
3. (a) Define addition of cardinal numbers and prove $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$, where α, β, γ are any three cardinal numbers.
(b) Define the concept of ordinal number and illustrate it by a suitable example.
4. (a) Define a group and prove that identity element in a group is unique.
(b) If a and b are any two elements of a group G then prove that $(ab)^{-1} = b^{-1} a^{-1}$, where a^{-1}, b^{-1} and $(ab)^{-1}$ respectively stand for inverses of a, b and ab in G .
5. (a) Define centre of a group and prove that centre of a group G is a normal subgroup of G .
(b) If G is a group and $a, b \in G$ then prove that $(ab)^2 = a^2 b^2$ if and only if G is abelian.
6. Define homomorphism of a group. State and prove fundamental theorem of homomorphism groups.
7. Define transpose of a matrix. If A and B are any two square matrices of the same order then prove that $(AB)' = B'A'$, where A', B' and $(AB)'$ respectively denote the transposes of A, B and AB . LNMUonline.com
8. Find the characteristic equation of the matrix :

$$A = \begin{bmatrix} 6 & -2 & 2 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Also find the eigen value of A .

9. Show that the system linear equations :

$$\begin{aligned} 5x + 3y + 7z &= 4 \\ 3x + 26y + 2z &= 9 \\ 7x + 2y + 10z &= 5 \end{aligned}$$
 are consistent and solve them.
10. (a) If $\alpha_1, \alpha_2, \dots, \alpha_n$ are the roots of the equation $x^n - 1 = 0$, then prove that $(1 - \alpha_1)(1 - \alpha_2) \dots (1 - \alpha_n) = 0$.
(b) Prove that in an equation with real coefficients, imaginary roots occur in conjugate pairs.
11. (a) Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ be in H.P.
(b) If α, β, γ are the roots of the equation $x^3 + p^2 + qx + r = 0$ then find the value of the symmetric function $\sum \alpha^2 \beta$ of the roots of the given equation.
12. Solve the equation $x^3 - 9x + 28 = 0$ by Cardon's method.