Mathematics (Sub./Gen.) (Arts/Sc.)

Answer eight questions in all, selecting at lest one from each Group.

Group-A

1. Prove that the following :
   (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   (b) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
2. Define equivalence relation and show that th relation '<' in the set of integers is not an equivalence relation.
3. (a) If a and b are two elements of a group G prove that $(ab)^{-1} = b^{-1} a^{-1}$.
   (b) Prove that identity element in a group is unique.
4. Define a field. Show that the set of all real numbers forms a field with respect to usual addition and multiplication.
5. Find the inverse of the matrix :

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 9 \end{bmatrix}$$

6. (a) If A be any square matrix, then show that :
   (i) $A + A'$ is symmetric.
   (ii) $A - A'$ is skew symmetric.
   (b) Find the matrices A and B when

$$A + B = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 2 & 2 \\ 1 & 1 & 3 \end{bmatrix} \text{ and } A - B = \begin{bmatrix} 1 & 4 & 4 \\ 4 & 2 & 0 \\ -1 & -1 & 2 \end{bmatrix}$$

7. (a) Define a subspace of a vector space V over the field F.
   (b) Prove that the intersection of two subspace of a vector space is subspace.

Group-B

8. If the sequence $\{a_n\}$ converges to l, then the $\{x_n\}$ sequence where $x_n = a_1 + a_2 + a_3 + ... + a_n$ also converges to l.
9. (a) State and prove D' Alembert's ratio test for the convergence of an infinite series.
   (b) Test the convergence of the series whose general term is $\sqrt{n^2 + 1} - n$.
10. (a) State and prove Cauchy's root test.
    (b) State and prove the Leibniz's test for the convergence of an alternating series.
11. Define continuity and differentiability of a function at a point. Show that a function differentiable at a point is necessarily continuous at the point.

Group-C

12. (a) Define radical axis and obtain equation of the radical axis of two given circles.
    (b) Find the value of k for which the circles $x^2 + y^2 + 5x + 3y + 7 = 0$ and $x^2 + y^2 - 8x + y + k = 0$ are orthogonal.
13. Find the equation of ellipse in standard form.
14. Define a parabola and obtain its equation in the standard form.
15. (a) If l, m, n are direction cosines of a line, then prove that $l^2 + m^2 + n^2 = 1$.
    (b) Find the angles between two straight lines whose direction cosines are (l_1, m_1, n_1) and (l_1, m_2, n_2).
16. Find the equation of the plane in intercept form.

—0—