Mathematics (Hons.) Paper-IV

Answer any six questions.

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1. Solve any two of the following differential equations :

(a) $\frac{dy}{dx} = e^{x+y} + x^2 e^y$

(b) $x \cos\left(\frac{y}{x}\right)(ydx + xdy) = y \sin\left(\frac{y}{x}\right)(xdy - ydx)$

(c) $\frac{dy}{dx} + \frac{y \log y}{x} = \frac{y}{x^2}(\log y)^2$

(d) $(x^2y - 2xy^2)\, dx - (x^3 - 3x^2y)\, dy = 0$

2. Solve any two of the following :

(a) $p^2 - p(e^x + e^{-x}) + 1 = 0$ (b) $(px - y)(x - py) = 2p$

(c) $y = (1 + p)x + ap^2$ (d) $p^2y + 2px = y$

3. Solve any two of the following differential equations :

(a) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x$ (b) $\frac{d^2y}{dx^2} - y = x^2 \cos x$

(c) $\frac{d^2y}{dx^2} + a^2y = \sec ax$ (d) $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$

4. (a) Prove that the system of confocal conics $\frac{x^2}{(a^2+\lambda)} + \frac{y^2}{(b^2+\lambda)} = 1$ is self-orthgonal.

(b) Find the orthogonal trajectories of the cardiodes $r = a(1 - \cos\theta)$, where a is the parameter.

5. (a) Solve the following differential equation $x^2\frac{d^2y}{dx^2} - (x^2 + 2x)\frac{dy}{dx} + (x + 2)y = x^3e^x$ by using the method of variation of parameters.

(b) Solve any one of the following partial differential equations :

(i) $y^2p + x^2q = x^2y^2z^2$ (ii) $(x^2 - yz)p + (y^2 - zx)q = (z^2 - xy)$

6. Apply Charpit's method to solve any one of the following differential equations for complete integral.

(a) $p^2 + q^2 - 2px - 2qy + 1 = 0$ (b) $(p^2 + q^2)y = qz$

7. (a) Prove that the orthogonality relation for Legender polynomials

$$\int_{-1} [P_n(x)]^2\, dx = \frac{2}{2n+1}.$$

(b) Show that the recurrence relation for Legender polynomials

$nP_n = (2n - 1)x P_{n-1} - (n - 1)P_{n-2}$; $n \geq 2$.

8. Prove the following recurrence relations for Bessel's functions :

(a) $\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x)$

(b) $J'_n(x) = J_{n-1}(x) - \left(\frac{n}{x}\right)J_n(x)$

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9. (a) Prove that :

$$F(\alpha; \beta; x) = \frac{(\beta)}{\Gamma(\alpha)\Gamma(\beta - \alpha)} \int_0 (1 - t)^{\beta - n - 1} t^{\alpha - 1} e^{xt}\, dt, \text{ where } \beta > \alpha > 0$$

(b) Show that if $|x| < 1$ and $|x/(1 - x)| < 1$ then

$$F(\alpha; \beta; \gamma; x) = (1 - x)^{-\alpha} F\left[\alpha, \gamma - \beta; \gamma; \frac{-x}{1 - x}\right]$$

10. (a) If $L\{F(t)\} = f(s)$ then prove that $\frac{1}{s} f(s) = L\left\{\int_0^t f(u)\, du\right\}$.

(b) Find : (i) $L(t^3 e^{-3t})$ (ii) $L(\cos^2 at)$

11. (a) Find the Laplace transform of $\frac{\sin at}{t}$. Does the transform of $\frac{\cos at}{t}$ exist ?

(b) Find the Laplace transform of $\int_0^t \left(\frac{1-e^{-2x}}{x}\right) dx$.

12. Apply Laplace transform to solve :

(a) $\frac{d^2y}{dt^2} + t\frac{dy}{dt} - y = 0$ if $y(o) = 0$, $\frac{dy}{dt} = 1$, if $t = 0$

(b) $\frac{d^2y}{dt^2} + y = 6\cos 2t$ if $y = 3$, $\frac{dy}{dt} = 1$, when $t = 0$

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