Mathematics (Hons.) Paper-VII (Sc. & Arts)

Answer any six questions.

1. (a) Obtain the general conditions of equilibrium of a system of forces acting inone plane upon a rigid body.
(b) Three forces P, Q, R act along the sides BC, AC, BA of an equilateral triangle ABC. If their resultant is a force parallel to BC through the centroid of the triangle, prove that $Q = R = \frac{1}{2} P$.

2. (a) State and prove the converse of the principle of virtual work.
(b) The middle points of the opposite sides of a jointed quadrilateral are connected by light rods of lengths L and L'. If T and T' be the tensions in these rods, prove that if the system is in equilibrium then $\frac{T}{L} + \frac{T'}{L'} = 0$

3. (a) Define common catenary and obtain its equation in the form of $y = c \cosh \left(\frac{x}{c}\right)$.
(b) A uniform chain of length $2l$ has its extremitics fastened to two fixed points at the same level and the sag in the middle is h. Prove that the span is $\frac{l^2 - h^2}{h} \log \frac{l+h}{l-h}$

4. (a) Establish the energy test for stability.
(b) A body rests in equlibrium upon another fixed body the portions of the two bodies in contact being spheres of radii r and R respectively and the straight line joining the centres of the sphere being vertical, if the first body be slightly displaced, find whether the equilibrium is stable or unsstable, the body being rough enough to prevent sliding.

5. (a) Show that two simple Harmonic Motions of the same period and in the same straight line may be compounded.
(b) A particle starts with a given velocity v and moves under a retardation equal to k times the space described. Show that the distance traversed before it comes to rest is $\frac{v}{\sqrt{k}}$.

6. (a) A particle moves in a plane with acceleration which is always directed to a fixed point in the plane. Obtain the differential equation of the path.
(b) In an orbit described under a force to a centre, the velocity at any point is inversely proportional to the distance of the point from the centre of force. Show that the path is an equiangular spiral.

7. (a) Prove that the work done against the tension in stretching a light elastic string is equal to the product of its extension and the mean of its initial and final tension.
(b) A light spring is kept compressed by the action of a given force, the force is suddenly reversed, prove that the greatest subsequent extension of the spring is three times its initial contraction.

8. (a) Find the minimum time of oscillation of a compound pendulum.

(b) Define center of suspension and center of oscillation of a compound pendulum and show that the two can be interchanged.

9. (a) Show that the necessary and sufficient condition for three non-parallel, non-zero vectors $\vec{a}, \vec{b}, \vec{c}$ be coplanar is $[\vec{a}, \vec{b}, \vec{c}] = 0$

(b) Show that :

$$(\vec{b} \times \vec{c}) \cdot (\vec{a} \times \vec{d}) + (\vec{c} \times \vec{a}) \cdot (\vec{b} \times \vec{d}) + (\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 0.$$

10. (a) Prove that a necessary and sufficient condition for the vector $\vec{u}$ of a scalar variable t to have constant magnitude is $\vec{u} \cdot \frac{d\vec{u}}{dt} = 0$.

(b) Prove that :

$$\frac{d}{dt}[\vec{r_1}\,\vec{r_2}\,\vec{r_3}] = \left[\frac{d\vec{r_1}}{dt}\,\vec{r_2}\,\vec{r_3}\right] + \left[\vec{r_1}\,\frac{d\vec{r_2}}{dt}\,\vec{r_3}\right] + \left[\vec{r_1}\,\vec{r_2}\,\frac{d\vec{r_3}}{dt}\right]$$

11. (a) Define divergence and curl of a vector function $\vec{v}$ prove that div (curl $\vec{v}$) = 0.

(b) Prove that :

$$\nabla \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\nabla \times \vec{a}) - \vec{a} \cdot (\nabla \times \vec{b})$$

12. (a) State and prove Gauss theorem.

(b) Prove that $\nabla \times \vec{r} = 3$ where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.