HG(1) — M (2) Sc. &
Arts – New

2018

Time : 3 hours

Full Marks : 90

Pass Marks : 28

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Answer any **six** questions.*

1. (a) Prove that every Cartesian equation of second degree represents a conic.

   (b) Prove that in general a straight line cuts a conic in two points real or imaginary.

2. (a) Define a confocal system of conics. Prove that two conics of a confocal system pass through any given point. Further prove that one of these conics is an ellipse and the other a hyperbola.

   (b) Prove that the locus of the pole of a given straight line with respect to a series of confocal conics is a straight line.

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3. (a) Obtain the polar equation of a conic in the standerd form $\frac{\ell}{r} = 1 + e\cos\theta$, where ℓ is the semi latus rectum, e the eccentricty of the conic and focus of the conic taken as the pole.

   (b) Obtain the equation of the director circle of the conic $\frac{\ell}{r} = 1 + e\cos\theta$.

4. (a) Prove that the polar equation of the circle circumscribing the triangle formed by the tangents to the conic $\frac{\ell}{r} = 1 + \operatorname{cosec}\theta$ at the points whose vectorial angles are 2α, 2β, 2γ is :

   $2r\cos\alpha\cos\beta\cos\gamma = \ell\cos(\theta - \alpha - \beta - \gamma)$.

   (b) Find those diameters of the conic $S = x^2 + 4xy + y^2 - 2x - 2y - 6 = 0$ which touch the parabola $y^2 = 8x$.

5. (a) Find the volume of the tetrahedron whose vertices are (x_r, y_r, z_r), $r = 1, 2, 3, 4$, the axes being rectangular.

   (b) Find the volume of the tetrahedron formed by the planes $\ell x + my + nz = p$, $\ell x + my = 0$, $my + nz = 0$ and $\ell x + nz = 0$.

6. (a) Find the condition that the line $\frac{x-\alpha}{\ell} = \frac{y-\beta}{m} = \frac{x-\gamma}{n}$ may lie in the plane $ax + by + cz + d = 0$.

(b) Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar.

7. (a) Define a cone and obtain the equation of tangent plane to a cone a given point of the cone.

(b) Prove that the cone $yz + zx + xy = 0$ is cut by the plane $px + qy + rz = 0$ is perpendicular lines if $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 0$.

8. (a) Define a central conicoid. Find the condition that the plae $\ell x + my + nz = p$ may touch the conicoid $ax^2 + by^2 + cz^2 = 1$.

(b) A tangent plane to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ meets the coordinate axes in points A, B and C. Prove that the centroid of the triangle ABC lies on the locus of $\frac{a^2}{x^2} + \frac{b^2}{y^2} + \frac{c^2}{z^2} = 9$.

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9. (a) Expand $\cos\theta$ in an infinite series of ascending power of θ.

(b) Solve $x^n = 1$. Show that the sum of its roots is zero. Hence or otherwise deduce that it has two or one real roots according as n is even or odd.

10. (a) Prove that $\log(\alpha + i\beta) = \frac{1}{2}\log(\alpha^2 + \beta^2) + i\tan\frac{\beta}{\alpha}$.

(b) Express $(\alpha + i\beta)^{x+iy}$ in the form $A + i\beta$.

11. (a) Prove that :

$$\frac{1}{2}\tan\frac{\theta}{2} + \frac{1}{2^2}\tan\frac{\theta}{2^2} + \frac{1}{2^3}\tan\frac{\theta}{2^3} + \ldots\ldots = \frac{1}{\theta} - \cot\theta.$$

(b) Obtain the sum of the series :

$$1 + \frac{\cos\alpha}{\cos\alpha} + \frac{\cos 2\alpha}{\cos^2\alpha \cdot \underline{|2}} + \frac{\cos 3\alpha}{\cos^3\alpha \cdot \underline{|3}} + \ldots\ldots \text{ to}$$

adnif.

12. (a) Express $\cos\theta$ as an infinite product.

(b) If $\cos hx = \sec\theta$, prove that :

$$i\theta = \log_e \tan\left(\frac{\pi}{4} + \frac{ix}{2}\right).$$