**HG(3) — Math (6)**
**Sc. & Arts**

Course – B.Sc Part-III
Subject – Maths
Paper – 6

# 2020

*Time : 3 hours*

*Full Marks : 90*

*Pass Marks : 41*

*Candidates are required to give their answers in their own words as far as practicable.*

*The questions are of equal value.*

*Answer any **six** questions.*

1. (a) Prove that the set of all automorphisms of a group forms a group with respect to the composite composition.

   (b) Prove that for an abelian group, the only inner automorphism is the identity mapping whereas for non abelian groups there exist non trivial automorphism.











2. (a) Prove that every group of prime order is cyclic.

   (b) Prove that if G is a group of order $p^n$ then its centre $z \neq \{e\}$. Where p is a prime order.

3. State and prove Sylow's theorem on groups.

4. (a) State and prove fundamental theorem of homomorphism of ring.

   (b) Prove that the Kernel of a homomorphism of a ring R in to a ring S is an ideal in R.

5. Prove that every integral domain can be embedded in to a field.

6. State and prove Eisenstein criterion for irreducibility of a polynomial.

7. (a) Prove that necessary and sufficient condition for a non empty subset w of a vector space v(F) to be a subspaces of v is $a, b \in F$, $\alpha, \beta \in w \Rightarrow a\alpha + b\beta \in w$.

   (b) Prove that intersection of any two sub-spaces of a vector space is a subspace.

8. (a) If $w_1$ and $w_2$ are subspaces of a finite dimensional vector space v(F), then prove that $\dim(w_1 + w_2) = \dim(w_1) + \dim(w_2) - \dim(w_1 \cap w_2)$.

(b) Show that the set S = {(1, 2, 1), (3, 1, 5), (3, – 4, 7)} is linearly dependent when $S \leq V_3(R)$.

9. Let U and V be vector spaces over the field F and let T be a linear transformation from U in to V suppose that U is finite. Then prove that Rank(T) + nullity (T) = dim U.

10. (a) Prove that the characteristics roots of a real Symmetric Matrix are all real.

(b) Find the eigen values of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

11. (a) State and prove Schwarz's inequality for an inner product space.





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(b) If α, β are vectors in an inner product space V then prove that :

$$||\alpha + \beta|| \leq ||\alpha|| + ||\beta||$$

12. (a) If A and B are two submodules of an R-module M with $B \subseteq A$. Then prove that M/A is isomorphic to some quotient module of M/B.

(b) Prove that Kernel of homomorphism of module is submodule.

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