HG(I) — M (1)
Sc. & Arts

2020

Time : 3 hours

Full Marks : 90

Pass Marks : 28

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

*Answer any **six** questions.*

1. (a) Define countable and uncountable sets and prove that the set of rational numbers is countable.

   (b) Define addition of cardinal numbers and prove that $\alpha + (\beta + y) = (\alpha + \beta) + y$, where α, β and γ are any three cardinal numbers.

2. (a) If f is a similarity map of the well ordered set $(X, \leq)$ onto the subset $Y \subseteq X$ then prove that $x \leq f(x)$ for all $x \in X$.

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   (b) Prove that any well ordered set $(X, \leq)$ is totally ordered.

3. State and prove the well ordering thorem and deduce axiom of choice from well ordering theorem.

4. (a) Define a group with an example and prove that $(ab)^{-1} = b^{-1} a^{-1}$, $\forall\, a, b \in G$ where G be a group.

   (b) Prove that a necessary and sufficient condition for a non-empty subset H of a group G to be a subgroup is that $a \in H$, $b \in H \Rightarrow ab^{-1} \in H$, where b^{-1} is the inverse of b in G.

5. (a) State and prove Lagrange's theorem for a group.

   (b) If G is a group and $a, b \in G$, then prove that $(ab)^2 = a^2b^2$ if and only if G is abelian.

6. (a) State and prove the fundamental theorem of homomorphism of groups.

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(b) Prove that the intersection of two normal subgroups of a group G is also a normal subgroup of G.

7. (a) Define symmetric and skew symmetric matrices. Prove that every square matrix can be expressed uniquely as a sum of a symmetric and skew-symmetric matrix.

(b) If A and B are Hermitian matrices, show that AB + BA is Hermitian and AB – BA is skew Hermitian.

8. (a) Find the rank of the matrix $\begin{bmatrix} 2 & 1 & 4 & 7 \\ 3 & 6 & 2 & 1 \\ 0 & 0 & 1 & 5 \end{bmatrix}$.

(b) Prove that a system of non-homogenous equation AX = B is consistent if and only if rank of the coefficient matrix A is equal to the rank of the augmented matrix [A B].

9. (a) Show that the equations $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$ are consistent and solve them.

(b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$ and use the result to obtain A^{-1}.

10. (a) Prove that every equation of nth degree has n roots and no more.

(b) Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ are in G. P.

11. (a) If α, β and γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of $\Sigma\alpha^2\beta$.

(b) Reduce the cubic equation $a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$ to the form $z^3 + 3Hz + b = 0$, where symbols have their usual meanings.

12. (a) Discuss Cardon's method of solving the cubic equations.

(b) Solve by Ferrari's method $x^4 + 2x^3 - 7x^2 - 8x + 12 = 0$.

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