HG(I) — M (2)

Sc. & Arts

2020

Time : 3 hours

Full Marks : 90

Pass Marks : 28

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer any ***six*** *questions.*

1. (a) Prove that every Cartesian equation of second degree represents a conic section.

   (b) Prove that in general a straight line cuts a conic in two points real or imaginary.

2. Trace the conic $x^2 - 4xy + 4y^2 - 32x + 4y + 16 = 0$.

3. (a) Define a confocal system of conics. Prove that through every point in the plane of ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ two confocal conics can be drawn, one an ellipse and the other a hyperbola.

   (b) Show that the locus of the pole of a given straight line with respect to a system of confocal conics is a straight line which is normal to that confocal which the straight line touches.

4. (a) Obtain equation of normal to conic $\frac{l}{r} = 1 + l\cos\theta$ at the point whose vectorial angle is α.

   (b) If normals at α, β, γ on parabola $\frac{l}{r} = 1 + \cos\theta$ meet at point (ρ, ϕ), prove that $2\phi = \alpha + \beta + \gamma$.

5. (a) Prove that the general equation of first degree equation in x, y, z represents a plane.

   (b) Prove that the four planes $my + nz = 0$, $nz + lx = 0$, $lx + my = 0$ and $lx + my + nz = p_3$ form a tetrahedron whose volume is $\frac{p}{lmn}$.

6. (a) Find the condition that the two lines

$$\frac{x-\alpha_1}{l_1} = \frac{y-\beta_1}{m_1} = \frac{z-\gamma_1}{n_1} \text{ and}$$

$$\frac{x-\alpha_2}{l_2} = \frac{y-\beta_2}{m_2} = \frac{z-\gamma_2}{n_2} \text{ may be coplanar.}$$

(b) Find the length and equations of the line of shortest distance between the lines

$$\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2} \text{ and } \frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}.$$

7. (a) Find equation of tangent plane to sphere $x^2 + y^2 + z^2 = a^2$ at any point (x_1, y_1, z_1) on it.

(b) Show that the angle between the lines given by $x + y + z = 0$ and $ayz + fzx + cxy = 0$ is $\frac{\pi}{2}$ if $a + b + c = 0$ but $\frac{\pi}{3}$ if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.

8. (a) Find equation of normal at point (x_1, y_1, z_1) on the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

(b) If the axes be rectangular, find the locus of

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equi-conjugate diameters of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

9. (a) Expand $\sin\theta$ in ascending powers of θ.

(b) Using De Moiver's theorem solve the equation $x^7 + x^4 + x^3 + 1 = 0$.

10. (a) Prove $\log(\alpha + i\beta) = \frac{1}{2}\log(\alpha^2 + \beta^2) + i\tan^{-1}\frac{\beta}{\alpha}$.

(b) If $i^{i^{\cdot^{\cdot^{\text{to}\infty}}}} = \alpha + i\beta$, then prove that $\tan\frac{\pi\alpha}{2} = \frac{\beta}{\alpha}$ and $\alpha^2 + \beta^2 = e^{-\pi\beta}$.

11. (a) Find the sum of cosines of n angles which are in A.P.

(b) Sum to n terms the series $\tan^{-1}\frac{1}{1+1+1^2} + \tan^{-1}\frac{1}{1+2+2^2} + \tan^{-1}\frac{1}{1+3+3^2} + \cdots\cdots\cdots$.

12. (a) Express $\cos\theta$ as an infinite product.

(b) Show $2i\tan^{-1}\left\{i\tan\left(\frac{\pi}{4} - \theta\right)\right\} = \log_e \tan\theta$.

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