Mathematics (Hons.) Paper-I (Sc./Arts)

Answer six questions.

1. State and prove Schroeder-Bernstein theorem.
2. (a) In a partially ordered set $(X, \leq)$ prove that the following conditions are equivalent :
   (i) Every non-empty subset of X bounded above has a least upper bound.
   (ii) Every non-empty subset of X bounded below has a greatest lower bound.
   (b) Let $S = \{1, 2, 3, 4, 13\}$ and "$\leq$" be a binary relation defined on S by $x \leq y$ iff x divides y. Find maximal elements of the partially ordered set $(S, \leq)$.
3. (a) Prove that an infinite well ordered set has a subset with ordinal number w.
   (b) State and prove well-ordering theorem and assuming the truth of well-ordering theorem also prove the axiom of choice.
4. (a) Prove that the inverse element of an element in a group is unique.
   (b) Prove that the order of a cyclic group is equal to the order of its generator.
5. (a) The necessary and sufficient condition that a non-empty subset H of a group G be a subgroup is :
   $$a \in H,\ b \in H \Rightarrow ab^{-1} \in H$$
   (b) Prove that if H be a subgroup of the finite group G then the order of G is a multiple of the order of H.
6. (a) Define ring without unity with examples and prove that a ring R is without divisor of zero if and only if the cancellation laws for multiplication hold in R.
   (b) Show by an example that a ring may not be necessarily a field.
7. (a) If A and B are two non singular matrices of the same order then prove that $(AB)^{-1} = B^{-1}A^{-1}$.
   (b) Prove that any square matrix can be expressed uniquely as the sum of a symmetric and skew-symmetric matrix.
8. Reduce the matrix $A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 3 & 1 & 1 & 3 \end{bmatrix}$ to its normal form and hence determine its rank.
9. Find the eigen values of the following matrix :
   $$P = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & 6 \\ -1 & -2 & 0 \end{bmatrix}$$

10. (a) Show tht the equation : $\frac{A^2}{(x-a)} + \frac{B^2}{(x-b)} + \frac{C^2}{(x-c)} + \ldots + \frac{L^2}{(x-l)} = x - m$
    where a, b, c, ... are numbers different from one another, can not have an imaginary root.
    (b) If the roots of the equation $x^4 - 5x^3 + 11x^2 - 13x + 6 = 0$ be connected by the relation then find all its roots.
11. (a) Find the condition that the roots of the biquadratic $x^4 + px^3 + qx^2 + rx + s = 0$ may be in geometrical progression.
    (b) Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ are in H.P.
12. (a) Solve $x^3 + x^2 - 16x + 20 = 0$ by Cardon's method.
    (b) If two roots of Euler's cubic vanish, show that the biquadratic has a pair of equal roots given by $\frac{-a_1 \pm \sqrt{(-3H)}}{a_0}$