# Mathematics (Hons.) Paper-II (Sc. & Arts)

*Answer any six questions.*

1. Find the equation of a pair of tangents from an external point $(x_1, y_1)$ to the conic represented by the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.
2. (a) Prove that two confocal conics intersect at right angles.
   (b) Prove that one and only one conic of a confocal system will touch a given straight line.
3. (a) Obtain the polar equation of a conic in the standard form $\frac{l}{r} = 1 + e\cos\theta$, where $l$ is the semi latus rectum, e the ecentric and focus of the conic being taken as pole.
   (b) Show that the equations $\frac{l}{r} = 1 + e\cos\theta$ and $\frac{l}{r} = -1 + e\cos\theta$ represent the same conic.
4. Find the polar equation of normal to the conic $\frac{l}{r} = 1 + e\cos\theta$ the point $\theta = \alpha$..
5. (a) Find the length and equations of the line of shortest distance between two skew lines.
   (b) The intercepts made by a plane made on the coordinate axes are in the ratio 2 : 3 : 4. If the plane passes through (5, 0, –2), find the equation of the plane.
6. (a) Find the condition that two given straight lines may be coplanar.
   (b) Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanor.
7. (a) Find the condition that a given plane $ax + by + cz + d = 0$ may touch the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d_1 = 0$.
   (b) A sphere of radius r passes through origin and meets the axes in A, B and C. Prove that the centroid of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4r^2$.
8. Define a Central Conicoid. Find the condition that the plane $lx + my + nz = p$ may touch the conicoid $ax^2 + by^2 + cz^2 = 1$. 
9. (a) State and prove De Moivre's theorem for a rational index.
   (b) Obtain the $n^{th}$ roots of unity.
10. (a) Prove that $\log(\alpha + i\beta) = \frac{1}{2}\log(\alpha^2 + \beta^2) + i\tan^{-1}\frac{\beta}{\alpha}$.
    (b) Express $(\alpha + i\beta)^{x+iy}$ in the form of A + iB.
11. (a) Sum to n terms the series $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \ldots\ldots + \tan^{-1}\frac{1}{1+n(n+1)}$
    (b) Sum the series $\sin\theta - \frac{\sin 2\theta}{\underline{|2}} + \frac{\sin 3\theta}{\underline{|3}} - \ldots\ldots$ to inf.
12. Express $\sin\theta$ as an infinite product.