Mathematics (Hons.) Paper-II (Sc./Arts)

Answer any six questions.

1. Prove that the general equation of the second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a conic.

2. (a) Show that through every point in the plane of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, two confocal conics can be drawn, one an ellipse and the other a hyperbola.

   (b) Show that the locus of the pole of a given straight line with respect to a system of confocal conics is a straight line which is normal to that confocal which the straight line touches.

3. (a) Find the equation of the tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$ at the point whose vectorial angle is α.

   (b) Find the equation of the director circle of the conic $\frac{l}{r} = 1 + e\cos\theta$.

4. (a) Show that the line $\frac{l}{r} = A\cos\theta + B\sin\theta$ may be tangent to :

   $\frac{l}{r} = 1 + e\cos(\theta - \gamma)$ if $(A - e\cos\gamma)^2 + (B - e\sin\gamma)^2 = 1$

   (b) A circle passes through the focus S of the conic and meet it in four points whose distances from S are r_1, r_2, r_3 and r_4, prove that :

   $$\sum \frac{l}{r_1} = \frac{2}{l} \text{ and } r_1r_2r_3r_4 = \frac{d^2l^2}{e^2}$$

   where $2l$ is the latus rectum, e is the ecentricity and d is the diameter of the circle.

5. (a) Find the equation of the plane in intercept form.

   (b) Find the condition that the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ may lie in the plane $ax + by + cz + d = 0$.

6. (a) Find the volume of a tetrahedron whose vertices are (x_r, y_r, z_r), $r = 1, 2, 3, 4$, the axes being rectangular.

   (b) The length of two opposite edges of a tetrahedron are a, b their shortest distance is d and the angle between them is θ, prove that the volume is $\frac{1}{6}abd\sin\theta$.

7. (a) Prove that the spheres :

   $S = x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

   and $S' = x^2 + y^2 + z^2 + 2u'x + 2v'y + 2w'z + d' = 0$ intersect orthogonally if :

   $2uu' + 2vv' + 2ww' = d + d'$.

   (b) A plane passes through a fixed point (a, b, c) and cuts the axes at A, B, C. Show that the locus of the centre of the sphere OABC is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$.

8. Show that from an external point (α, β, γ), six normals can be drawn to an ellipsoid which lie on the cone of second degree, three of whose generating liens are parallel to the axes of the ellipsoid.

9. (a) Expand cosx in ascending powers of x.

(b) Find the equation whose roots are the nth powers of the roots of the equations

$$x^2 - 2x\cos\theta + 1 = 0.$$

10. (a) State and prove Gregory's series for expansion of in ascending powers of $\tan\theta$ when $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

(b) If $x = \log\tan\left(\frac{\pi}{4} + \frac{y}{2}\right)$, prove that $y = -i\log\tan\left(\frac{ix}{2} + \frac{\pi}{4}\right)$.

11. (a) Solve $x^5 - 1 = 0$. Prove that the sum of the nth powers of the roots, n being an integer not divisible by 5, is zero.

(b) If $u = \log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$, show that $\tan h = \tan\frac{1}{2}\theta$.

12. (a) Find the sum of the series :

$$\sin\theta\cos\theta - \frac{1}{3}\sin^3\theta\cos^3\theta + \frac{1}{5}\sin^5\theta\cos^5\theta \ldots\ldots \infty$$

(b) Express $\cos\theta$ as an infinite product.