Mathematics (Hons.) Paper-III

Answer any six questions.

1. (a) If $y = x^n \log x$ and y_n.

(b) If, $e^{a \sin^{-1} x} = a_0 + a_1 x + a_2 x^2 + \ldots\ldots$, prove that $(n+1)(n+2)a_{n+2} = (n^2 + a^2)a_n$.

2. Show that :

$$\frac{1}{\lfloor n} \frac{d^n}{dx^n} [x^n (\log x)^n] = 1 + S_1 \log x + \frac{S_2}{\lfloor 2} (\log x)2 + \ldots + \frac{S_n}{\lfloor n} (\log x)^n$$

where S_r is the suru of the products r at a time of the first n natural numbers.

3. (a) If $u = \sin^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}}$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$.

(b) If $u = cosec^{-1} \sqrt{\frac{x^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}}}$, show that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12} \left(\frac{13}{12} + \frac{\tan^2 u}{12} \right)$$

4. (a) Establish the formula for the radius of curvature of a curve in p– ψ form

$$\rho = p + \frac{d^2 p}{d\psi^2}$$

(b) If the normals at the points (r_1, θ_1), (r_2, θ_2), and (r_2, θ_2), on the cardioide $r = a(1 + \cos\theta)$ are concurrent then show that :

$$\tan \frac{\theta_1}{2} + \tan \frac{\theta_2}{2} + \tan \frac{\theta_2}{2} + \tan \frac{\theta_1}{2} \tan \frac{\theta_2}{2} + \tan \frac{\theta_3}{2} = 0$$

5. Evaluate any two of the following :

(a) $\int x \sqrt{\frac{a-x}{a+x}}\, dx$ (b) $\int \frac{x^2+1}{x(x^2-1)} dx$ (c) $\int \frac{x^2 dx}{(x \sin x + \cos x)^2}$

6. Evaluate $\int_0^{\frac{\pi}{2}} \cos^n x\, dx$

7. (a) Prove that the loop of curve, $x = t^2,\ y = t - \frac{t^3}{3}$ is of length $4\sqrt{3}$.

(b) Find the volume of the solid generated by the revolution of the curve $y^2(2a - x) = x^2$ about its asymptote.

8. (a) Define the Gama function $\Gamma(m)$ and establish that $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, where B (m, n) is the Beta function corresponding to the parameters m and n supposed positive.

(b) Obtain the value of $\int_0^{\pi} \frac{\tan^{-1}(\alpha x)}{x(1+x^2)}$

9. Investigate the extreme values of a function of two variables at a point.

10. (a) Define monotonic increasing and monotonic decreasing sequences of real numbers. Pove that every monotomic increasing sequence of real numbers bounded above converges to its least upper bound.

(b) Let a sequence $\{x_n\}$ of real number be defined by $x_{n+1} = \sqrt{3x_n}$ for all $n > 1$ and $x_1 = 1$. Prove that the sequence is convergent and $\lim_{n \to \infty} x_n = 3$

11. State and prove Cauchy's Condensation Test for the convergence of a series. Apply the condensation to show that the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)p}$ converges if $p > 1$ and diverges if $0 < p < 1$.

12. (a) State and prove D'Alembert's ratio test.

(b) Test the convergence of the series whose nth term is $\frac{n^2}{n^3+1} \cdot x^{n-1}$

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