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Mathematics (Sub./Gen.) (Sc./Arts)

Answer eight questions, selecting at least one from each Group.

Group-A

1. (a) State and prove Leibnitz's theorem to find the nth derivative of a product of two functions..
   (b) Expand sinx into power series using Maclaurin's Theorem.
2. (a) State and prove Maclaurin's series.
   (b) If $y = \sin^{-1} x$, then show that :
   $$(1 - x^2)\ y_{n+2} - (2x + 1)\ y_{n+1} - n^2\ y_n = 0.$$
3. (a) Establish the formula $p = r \sin \phi$,, where the symbols have their usual meanings.
   (b) Find the radius of curvature of a curve in pedal form.
4. (a) Find the limit, when n tends to infinity, of the series :
   $$\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots + \frac{1}{n+n}$$
   (b) Prove that : $\int \tan^n x\, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\, dx$
5. Find the area of a loop of the following curve : $x^4 = a^2 (x^2 - y^2)$
6. (a) Prove that B (m, n) = B (n, m).
   (b) Evaluate $\iiint_R u^2 v^2 w\, du\, dv\, dw$, where R isthe region, $u^2 + v^2 \le 1, 0 \le w \le 1$.
7. Solve any two of the following differential equations :
   (a) $\frac{dy}{dx} = \sin (x + y)$ (b) $(x^2 - y^2) \frac{dy}{dx} = 2xy$ (c) $\frac{dy}{dx} = e^{x+y} + x^2 e^y$
8. (a) Solve any one of the following differential equations :
   (i) $p^2 + 2xp - 3x^2 = 0$ (ii) $y = 2px + p^2$
   (b) Find the orthogonal trajectories of the cardiodes , where a is the parameter.
9. Solve any two of the following differential equations :
   (a) $\frac{d^2y}{dx^2} + y = \sin 2x$ (b) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 + e^x + \cos 2x$
   (c) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = xe^{2x}$ LNMUonline.com

Group-B

10. (a) Define scalar product of three vectors and show that in the scalar triple product, the dot and cross can be inter-changed without changing the value of the result.
    (b) Prove that : $[\vec{a} + \vec{b},\ \vec{b} + \vec{c}\ \vec{c} + \vec{a}\ ] \equiv 2\ [\vec{a}, \vec{b}, \vec{c}\ ]$
11. (a) The necessary and sufficient condition for the vector function $\vec{a}\ (t)$ to be constant is that
    $$\frac{d\vec{a}}{dt} = 0.$$
    (b) If $\vec{r_1} = t^2 \vec{i} - t\vec{j} + (2t + 1)\vec{k}$ $\vec{r_2} = (2t - 1)\vec{i} + \vec{j} - t\vec{k}$ find $\frac{d}{dt}\left(\vec{r_1} \cdot \vec{r_2}\right)$ when t = 1.
12. (a) If ϕ and $\vec{a}$ are continuously differentiable scalar and vector point functions respectively, then prove that : $curl\ (\phi \vec{a}) = \phi\ curl\ \vec{a} + (grad\ \phi) \times \vec{a}$.
    (b) Prove that $div \vec{r} = 3$. LNMUonline.com

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Group-C

13. (a) Obtain the general conditions of equilibrium of a system of forces acting in one plane upon a rigid body.
(b) Show that any system of forces, acting in one plane upon a rigid body can be reduced to either a single force or a single couple.

14. (a) State and prove the principle of virtual work for any system of forces in one plane.
(b) The middle points of the opposite sides of a jointed quadrilateral are connected by light rods of lengths l and l'. If T and T' be tensions in rods, then prove that :

$$\frac{T}{T} + \frac{T'}{T'} = 0$$

15. A particle moves in a straight line OA starting from the rest at A and moving with an acceleration which is always directed towards O and varies as the distance from O, discuss the motion.

16. Obtain expressions for tangential and normal velocities and accelerations.

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