

Mathematics (Sub/Gen.) Sc. / Arts

Answer eight questions, selecting at least one from each Group.

Group-A

- State and prove Leibnitz Theorem on successive differentiation.
 - If $y = \sin^{-1} x$, prove that $(1 - x^2) y_2 - xy_1 = 0$. Prove also that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$.
- For the curve $\frac{2a}{r} = (1 - \cos \theta)$, prove that $p^2 = ar$, where symbols have their usual meanings.
 - Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x}$
- Find the maximum value of the function $x^{1/x}$.
 - Define curvature and radius of curvature of a curve at a point. Prove that the radius of curvature ρ of a curve $y = f(x)$ at a point (x, y) is given by $\rho = \frac{(1 + y_1^2)^{3/2}}{y_2}$ where $y_1 = \frac{dy}{dx}$ and $y_2 = \frac{d^2y}{dx^2}$. LNMUonline.com
- Evaluate $\int_0^{\pi/2} \sin^2 x dx$
 - Evaluate $\int_a^b e^{ax} dx$ from first principle.
- State and prove the theorem of perpendicular axes on moment of inertia.
 - Obtain the moment of inertia of a thin uniform rod of length $2a$ and mass M about an axis through the centre of the rod and perpendicular to the rod.
- Find the whole length of the loop of the curve $9ay^2 = (x - 2a)(x - 5a)^2$
- Solve any two of the following differential equations :
 - $\frac{dy}{dx} = \sin(x + y)$
 - $(3y - 7x + 7) dx + (7y - 3x + 3) dy = 0$
 - $x \frac{dy}{dx} + \frac{y^2}{x} = y$
- Solve any one of the following differential equations :
 - $y = 2px + p^2$
 - $(y - px)(p - 1) = p$
 - Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{a^2 + \lambda} = 1$, where λ is a parameter.
- Solve any two of the following differential equations :
 - $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 3y = 2e^{3x}$
 - $\frac{d^2y}{dx^2} + a^2y = \sec ax$
 - $\frac{d^2y}{dx^2} - 13 \frac{dy}{dx} + 12y = x$

Group-B

- If \vec{a}, \vec{b} and \vec{c} are any three vectors then prove that
 - $\left[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a} \right] = 2 \left[\vec{a}, \vec{b}, \vec{c} \right]$
 - $\left[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a} \right] = \left[\vec{a}, \vec{b}, \vec{c} \right]$
- Prove that the derivative of vector of constant length is perpendicular to the vector.
 - If $\vec{r} = a \cos t \vec{i} + a \sin t \vec{j} + t \vec{k}$ then obtain $\frac{d\vec{r}}{dt}, \frac{d^2\vec{r}}{dt^2}$ and $\left| \frac{d^2\vec{r}}{dt^2} \right|$.
- If \vec{a} and \vec{b} are vector functions of a scalar quantity then prove that :
 - $\text{curl}(\vec{a} + \vec{b}) = \text{curl} \vec{a} + \text{curl} \vec{b}$
 - $\text{div}(\text{Curl} \vec{a}) = 0$ LNMUonline.com

Group-C

- Find the equation of line of action of a single resultant of a system of coplanar forces acting at different points of a rigid body.
 - Three forces P, Q, R act along the sides of a triangle formed by the lines $x + y = 1, y - x = 1$ and $y = 2$. Find the equation of the line of action of their resultant.
- The middle points of the opposite sides of a jointed quadrilateral are connected by light rods of lengths l and l' . If T and T' be the tensions of these rods respectively, prove that $\frac{T}{l} + \frac{T'}{l'} = 0$
- A particle moves along a straight line with an acceleration which is always directed towards a fixed point in the straight line and is proportional to its distance from the fixed point. Determine the motion.
- Obtain the tangential and normal components of velocity and acceleration of a particle moving along a curve in a plane.