Mathematics (Hons.) Paper-VII (Sc. & Arts)

Answer any six questions.

1. (a) Find the equation of the line of action of the resultant of a system of coplanar forces acting on a rigid body.

   (b) Forces P, Q, R act along the sides of a triangle formed by the lines $x + y = 1$, $y - x = 1$ and $y = 2$. Find the magnitude of the resultant and its line of action.

2. (a) State and prove principle of virtual work for a system of coplanar forces.

   (b) Two equal uniform rods AB and AC each of length 2b, are jointed at A and rest on a smooth vertical circle of radius a. Show that if 2θ be the angle between them $b.\sin^3\theta = a\cos\theta$.

3. (a) For a common catenery prove that :

   (i) $x = c\log(\sec\psi + \tan\psi)$ and (ii) $T = w.y$.

   (b) A uniform chain of length l is to be suspended from two points A and C in the horizontal line so that either terminal tension is n times that of the lowest point.

   Show that the span AC must be $\dfrac{1}{\sqrt{(n^2-1)}}\log\left\{M + \sqrt{(n^2-1)}\right\}$

(a) Find the conditions of stability for a body with one degree of freedom.

(b) A solid sphere rests inside a fixed hemispherical bowl of radius twice the radius of the sphere. Show that the equilibrium is stable, how so ever large weight is attached to the sphere at its highest point.

5. (a) Define simple harmonic motion. Find its time period, amptitude and frequency.

(b) In a S.H.M. of amplitude a and period T, show that $\int_0^T v^2 dt = \frac{2\pi^2 a^2}{T}$

6. (a) A uniform extensible string of weight w and natural length l is suspended from a fixed point and to the other end is hung a weight w_1; if λ be the modulus of elasticity, show that the whole extension of the string is $\frac{l}{\lambda}\left(\frac{w}{2}+w_1\right)$.

(b) If v_1 and v_2 be the linear velocities of a planet when it is respectively nearest and farthest from the sun, then prove that $(1-e)v_1=(1+e)v_2$.

7. State and prove D'Alembert's principle and prove that the rate of change of momentum of a body in any given direction is equal to the resolved part of the external forces in the same direction.

8. (a) State and prove Kepler's law for central orbit.

(b) A particle describes the curve $r^n = a^n \cos n\theta$ under a force P to the pole. Find the law of force.

9. (a) Prove that : $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$

(b) Prove that : $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2[\vec{a}\vec{b}\vec{c}]$

10. (a) If $\vec{v}\times\frac{d\vec{v}}{dt}=0$ then show that $\vec{v}(t)$ is a constant vector.

(b) Evaluate : $\frac{d}{dt}\left(\vec{r}.\frac{d\vec{r}}{dt}\times\frac{d^2\vec{r}}{dt^2}\right)$

11. (a) Prove : Div. $(\phi \vec{a}) = \phi\, div \vec{a} + \vec{a}(grad.\phi)$

(b) Find div. $\vec{v}$ and curl $\vec{v}$ where $\vec{v} = \nabla(x^3 + y^3 + z^3 - 3xyz)$

12. State and prove Stoke's theorem.

—oo—