

Mathematics (Hons.) Paper-VII (Sc./Arts)

Answer six questions, selecting at least one from each Group.

Group-A LNMUonline.com

- (a) Find the equation of the line of action of the resultant of a system of coplanar forces.
(b) Three forces $3P$, $7P$ and $5P$ act along the sides AB , BC and CA of an equilateral triangle ABC . Find the magnitude direction and the line of action of the resultant.
- (a) State and prove the converse of the principle of virtual work.
(b) The middle points of the opposite sides of a jointed quadrilateral are connected by light rods of length L and L' . If T and T' be the tensions in these rods, prove that
$$\frac{T}{L} + \frac{T'}{L'} = 0.$$
- (a) A body rests in equilibrium upon another fixed body the portions of the two bodies in contact being spheres of radii r and R respectively and the straight line joining the centres of the spheres being vertical; if the first body be slightly displaced, find whether the equilibrium is stable or unstable, the bodies being rough enough to prevent sliding.
(b) Establish the energy test for stability.
- (a) Define common catenary and obtain its equation in the form $y = c \cosh\left(\frac{x}{c}\right)$.
(b) A string hangs under gravity and it is loaded so that the weight of each element of it is proportional to the horizontal projection of that element. Show that it will hang in the form of a parabola.

Group-B

- (a) A particle moves in a straight line so that its acceleration varies as the distance from the fixed point in the straight line and directed away from the fixed point. Discuss the motion.
(b) A particle starts with a given velocity v and moves under a retardation equal to k times the space described. Show that the distance traversed before it comes to rest is $\frac{v^2}{2k}$. LNMUonline.com
- (a) A particle moves in a plane with an acceleration which is always directed to a fixed point in the plane. Obtain the differential equation of the path.
(b) In an orbit described under a force to a centre, the velocity at any point is inversely proportionate to the distance of the point from the centre of force. Show that the path is an equiangular spiral.
- (a) State and explain Kepler's laws for central orbit.
(b) A particle moves in a path so that its acceleration is always directed to a fixed point and is equal to $\frac{\mu}{(\text{distance})^2}$; show that the path is a conic section with the fixed point as its focus.
- (a) Determine the motion of a rigid body acted on by the force of gravity only and moving about a fixed horizontal axis. LNMUonline.com
(b) Find the minimum time of oscillation of a compound pendulum.

9. (a) Prove that : $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2 [\vec{a} \vec{b} \vec{c}]$
- (b) Prove that : $(\vec{b} \times \vec{c}) \cdot (\vec{a} \times \vec{d}) + (\vec{c} + \vec{a}) \cdot (\vec{b} \times \vec{d}) + (\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 0$
10. (a) Prove that : $\text{curl}(\phi \vec{a}) = \phi \text{curl} \vec{a} + (\nabla \phi) \times \vec{a}$
- (b) Find $\text{div} \vec{v}$ and $\text{curl} \vec{v}$, where $\vec{v} = \nabla(x^3 + y^3 + z^3 - 3xyz)$.
11. (a) Define reciprocal vectors. Prove that :
- (i) $aa' = bb' = cc' = 1$
- (ii) $[a' b' c'] [abc] = 1$, where a, b, c are non-coplanar vectors
- (b) Find a set of vectors reciprocal to set of vectors.
- $2\vec{i} + 3\vec{j} - \vec{k}, \vec{i} - \vec{j} - \vec{k}, \vec{i} + 2\vec{j} + 2\vec{k}$.
12. Verify Gauss Theorem for the function $\vec{F} = x^2 \vec{i} + y \vec{j} + z \vec{k}$ over the unit cube.