2022

Time : 3 Hours

Maximum Marks : 90

D-112

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer any **Six** questions.

1. (a) State and prove the necessary and sufficient conditions of a bounded function defined on the interval $[a, b]$.

   (b) Show that $f(x) = x$ is R-integrable on $[1,2]$.

2. (a) If f is continuous on $[a, b]$, then show that f is R-integrable.

   (b) State and prove fundamental theorem of calculus using R-integrability.

3. (a) Discuss the convergence of $\int_0^{\pi/2} \log \sin x \, dx$.

   (b) State Dirichlet's test for convergence of an improper integral and hence test the convergence of the integral

   $$\int_a^\infty \frac{1}{\sqrt{x}} \sin x \, dx, a > 0.$$

4. State and prove Implicit function theorem.

5. (a) Obtain Cauchy-Riemann equations in polar form.

   (b) Prove that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4 \frac{\partial^2}{\partial z \, \partial \bar{z}}$

6. (a) Show that bilinear transformation transforms circles into circles.

   (b) Find a bilinear transformation which transforms the point $z = 2, 1, 0$ into $w = 1, 0, i$ respectively.

7. (a) Show that differentiability of a complex valued function $f(z)$ implies continuity. But converse is not true justify.

(b) Prove that the function $f(z) = |z|^2$ is nowhere differentiable except at the origin.

8. (a) Define inverse points with respect to circle and show that two points z_1, z_2 are inverse points with respect to circle $z\bar{z} + b\bar{z} + \bar{b}z + c = 0$ if $z_1\bar{z_2} + b\bar{z_2} + \bar{b}z_1 + c_1 = 0$.

(b) Show that the function $f(z) = xy + iy$ is not analytic.

9. (a) If (E, d) is a metric space then show that (E, ρ) is also a metric space where $\rho(x, y) = \frac{d(x,y)}{1+d(x,y)} \forall\, x, y \in E$.

(b) Show that an open sphere in a metric space (E, d) is an open set.

10. (a) Show that every convergent sequence (x_n) is a metric space (E, d) is a Cauchy sequence.

(b) Show that, In a complete metric space, every closed set is complete.

11. State and prove Baire's Category Theorem.

12. Prove that a metric space is compact if and only if it is sequentially compact.

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