2018-20(New)

Time : 3 hours

Full Marks : 70

Pass Marks : 31.5

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from all the Groups as directed.

Group – A

(Compulsory)

1. Indicate whether the following statements are True or False : 10×2 = 20

   (a) Arbitrary intersection of subspaces of a vector space is a null space.

   (b) If S and T are linear operators on a vector space U, then $(S+T)^2 = S^2 + 2ST + T^2$.

   (c) A matrix A is said to be in normal form if it can be written as $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

   (d) The product of two self-adjoint operators in an inner product space is always self-adjoint.

   (e) A bilinear form is reflexive iff it is either symmetric or skew-symmetric.

   (f) A bilinear form f on a vector space v is called degenerate iff for each α in V, $f(\alpha, \beta) = 0 \; \forall \beta \in V$.

   (g) If λ is a eigen value of T, then λ^{-1} is an eigen value of T^{-1}.

   (h) If A be a $n \times n$ matrix with eigen values zero. Then A is nilpotent.

   (i) If V(F) be an inner product space, then it is called a Euclidean space if F is the field of complex number.

   (j) If f, g are two bilinear form on $V \times V$ then $f+g$ is also bilinear form.

Group – B

Answer any four of the following : 5×4 = 20

2. (a) State and prove Cayley-Hamilton theorem.

(b) Find eigen vectors of $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

(c) Define similarity of linear transformations and show that the relation of similarity is an equivalence relation in the set of all linear transformations on a vector space V(F).

(d) If T_1 and T_2 be self-adjoint transformations of an inner product space V(F) then prove that $T_1 + T_2$ is also self-adjoint.

(e) Define nilpotent tranformation on a vector space V(F).

Group – C

Answer any **three** of the following : 10×3 = 30

3. (a) Define symmetric bilinear form and skew-symmetric bilinear form. If V is a finite-dimensional vector space, then prove that a bilinear form f on V is symmetric iff its matrix A in some ordered basis is symmetric.

(b) State and prove Sylvester's Theorem.

(c) Find the rank, signature and index of the following quadratic forms :

(i) [illegible] $3z^2$ $4yz + 6xz$

(ii) $x_1^2 \quad 2x_1x_3 + 2x_2^2 + 4x_2x_3 + 6x_3^2$

(d) State and prove primary decomposition theorem

(e) Define nilpotent linear transformation on a vector space V(F). If S and T are nilpotent linear transformations on V(F) which commute is ST = TS, prove that S + T and ST are also nilpotent.