2020(New)

Time : 3 hours

Full Marks : 70

Pass Marks : 31.5

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

*Answer from **all** the Groups as directed.*

Group – A

1. Examine whether the following statements are 'True' or 'False' : 2×10 = 20

(a) A complex number is said to be purely imaginary if its real part is zero.

(b) If $z = x + iy$ then $z + \bar{z} = 2x$.

(c) The value of $e^{2\pi i} = 1$.

(d) If L is a circle, the value of $\int_L \frac{dz}{z-2}$ is $2\pi i$.

(e) A contour is a chain of infinite number of regular arc.

(f) The series $f(z) = \sum_{n=\infty}^{\infty} a_n (z-z_0)^n$, where $a_n = \frac{1}{2\pi i}\int_C \frac{f(z)dz}{(z-z_0)n+1}$ represents Laurent series of f(z).

(g) An integral function is also called entire function.

(h) The residue of $\frac{1}{(z^2+1)^3}$ at $z = i$ is $\frac{3}{16i}$.

(i) A set of bilinear transformations with respect to composition of product form a Abelian Group.

(j) The value of $\int_0^\infty \frac{\sin x}{x} dx$ is $\frac{\pi}{2}$.

Group – B

Answer any **four** questions of the following : 5×4 = 20

2. If $W = f(z)$ is an analytic function of z such that $f'(z) \neq 0$, then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \log|f'(z)| = 0$.

3. Prove that $\left|\int_L f(z)dz\right| \le \int_L |f(z)||dz|$.

4. Determine the poles of the function $f(z) = \dfrac{z^2}{(z-1)^2(z+2)}$ and the residue at each pole.

5. Find bilinear transformation which maps the points $z_1 = 2$, $z_2 = i$, $z_3 = -2$ into the points $w = 1$, $w_2 = i$ and $w_3 = -1$.

6. Evaluate the residues of $\dfrac{z^3}{(z-1)(z-2)(z-3)}$ at $z = 1, 2, 3$ and infinity and show that their sum is zero.

Group – C

Answer any **three** questions : 10×3 = 30

7. Derive polar form of Cauchy-Riemann equations.

8. State and prove Liouville's Theorem.

9. State and prove Taylor's Theorem.

10. State and prove Rouche's Theorem.

11. If f(z) has a pole of order m at $z = z_0$, then the function ϕ defined Ly $\phi(z) = (z - z_0)^m f(z)$ has a removable singularity at z_0 and $\phi(z_0) \neq 0$.

 Also the residue at z_0 is given by $\dfrac{\phi^{(m-1)}(z_0)}{\lfloor(m-1)}$.