2020(New)

Time : 3 hours

Full Marks : 70

Pass Marks : 31.5

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from all the Groups as directed.

Group – A

1. State whether the following statements are 'True' or 'False' : 2×10 = 20

   (a) The length of interval $\{I_i\}$ is defined by $\underset{i}{U}I_i$.

   (b) The outer measure of an interval is its length.

   (c) If A and B are disjoint measurable subsets of [a, b] and if f is bounded L-integrable function on [a, b] then $\int_A f + \int_B f$ is equal to $\int_{A \cup B} f$.

   (d) If $\{f_n\}$ be a sequence of measurable function f on E then $\underset{n \to \infty}{\alpha L} \int_E f_n(x)dx = \int_E f(x)dx$.

   (e) Dini's Derivative $D_- f(x)$ is equal to $-D_-(-f(x))$.

   (f) If $f(x) = x \sin\dfrac{1}{x}; \quad x \neq 0$

   $\qquad = 0 \; ; \quad x = 0$

   then $D^+ f(0) = -1$.

   (g) p-Norm of any $f \in L^p(a, b)$ is defined by

   $\left[\displaystyle\int_a^b |f|\right]^{1/p}$.

   (h) If $0 < p < 1$ and f, g are non-negative function in $\lfloor p$, then $\|f + g\|_p \geq \|f\|_p + \|g\|_p$.

   (i) If μ is a σ-finite measure on R, then the extension $\overline{\mu}$ of μ to S^*, where μ^* be an outer measure on H(R) and S^* the class of μ^* measurable sets, is also σ-finite.

   (j) If μ is a measure on R, then μ is monotone and substractive.

Group – B

Answer any **four** questions of the following :

$$5×4 = 20$$

2. Prove that a continuous function defined on a measurable set is measurable.

3. If the Lebesgue integral of Non-negative measurable function f over [0, 1] be zero, show that f = 0 a. e.

4. Show that an indefinite integral is a continuous function.

5. Define measure and measure space with example.

6. Define distance function on L^P space and prove that (L^P, d) is a metric space.

Group – C

Answer any **three** questions of the following :

$$10×3 = 30$$

7. If $\{E_i\}_{i=1}^{n}$ is a family of disjoint measurable sets, then for any set $W \leq R^k$, prove that

$$m^*\left[W \cap \left\{\bigcup_{i=1}^{n} E_i\right\}\right] = \sum_{i=1}^{n} m^*\{W \cap E_i\}.$$

8. Prove the following integrability properties :

   (a) f and g are L-integrable $\Rightarrow$ f + g is L-integrable

   (b) f is L-integrable $\Rightarrow$ |f| is L-integrable and further $|\int f d\mu| \leq \int |f| d\mu$

   (c) $|f| \leq g$ and g is L-integrable $\Rightarrow$ f is also L-integrable

9. State and prove Lebesgue differentiation theorem.

10. State and prove extension theorem for measure.

11. State and prove Minkowski's inequality.

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